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An experimental study of the effects of wall conductivity, non-uniform magnetic fields and variable area ducts on liquid metal flows at high Hartmann number. Part 2. Ducts with conducting walls

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Results from experiments with four different ducts are reported when magnitudes of the field strength and mean velocity are such that the Hartmann number and interaction parameter are large. The first is a straight, circular, highly conducting wall duct situated in a non-uniform transverse magnetic field. Results suggest that as a first approximation the flow may be regarded as being fully developed throughout. In fact there is a slight distortion of the flow in the non-uniform field region revealed by hot-film probe measurements of the streamwise velocity which varies in a novel but readily explicable manner. The second duct is similar except that its wall is weakly conducting. A pressure drop across the non-uniform field region suggests that the behaviour of the flow is weakly reminiscent of that in a non-conducting duct. The two other ducts also have weakly conducting walls but contain either one or two 90° bends and are situated in a uniform field. Symmetry of each duct about its mid-point leads to symmetric potential distributions which indicate the existence of two symmetrically arranged recirculating current flows and these lead to pressure drops across the bends. In the duct with two bends, part of it, the offset, lies parallel to the field lines and a surprising prediction relating the pressure drop across the offset to N finds some support.

1. Introduction

Part 1 of this study (Holroyd 1979) was devoted to a thorough experimental investigation of MHD flow along straight non-conducting ducts situated in a transverse non-uniform magnetic field when both the Hartmann number $M = aB_0(\sigma/\eta)^{\frac{1}{2}}$ and interaction parameter $N = M^2/Re = aB_0^2\sigma/\rho V$ were large (B_0 = representative value of flux density, V = mean velocity of flow, a = duct radius, σ = conductivity of the fluid, η = viscosity, ρ = density and $Re = \rho Va/\eta$ = Reynolds number of flow). Circular and rectangular (nominally square) ducts were used but the former was subjected to a more detailed study because of its obvious practical applications and because there is a corresponding theoretical study available with which to compare the results (Holroyd & Walker 1978). The large values of M and N were necessary to

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simulate the inviscid inertialess flow to which the theory applies but the experimental results did not entirely agree with the predicted behaviour of the flow for three reasons:

(i) It was not possible to satisfy the condition $N \gg M^{\frac{3}{2}}$ for an inertialess flow – usually $N \approx M^{\frac{3}{2}}$;

(ii) even though $200 < M < 525$ viscous effects were not entirely absent from some parts of the flow;

(iii) because a region of non-uniform field gives rise to disturbances over $O(aM^{\frac{1}{2}})$ lengths upstream and downstream, it was inevitable, in some cases, that the effects of the non-uniform fields at the upstream and downstream ends of the magnet would overlap the similar effects in the region of interest between them.

Even so, it was observed that when the field strength changed from a uniform value B_0 to a uniform value $\frac{1}{2}B_0$ (i) a pressure drop of $O(aM^{\frac{1}{2}} \times$ fully developed flow pressure gradient) was introduced into the pressure distribution whose exact value varied with M and was slightly larger than the predicted value; (ii) associated with the pressure drop was a localized 'trough' in the pressure distribution whose peak value was larger than the pressure drop by a factor of 1·5–2; (iii) the flow was severely retarded near the centre of the duct in the non-uniform field region, the effect apparently being more pronounced as M increased, when $M \rightarrow \infty$ and $N \gg M^{\frac{3}{2}}$ theory predicts stagnant fluid and reverse flows.

Part 2 comprises the results of experiments with four different ducts of circular cross-section with conducting walls. Two of the ducts are straight and are placed in the non-uniform field used in part 1 (briefly described above) while the other two contain one or two 90° bends but are situated in a uniform transverse field. Although there are some relevant theoretical results available, care must be exercised when comparing them with the experimental results since the assumptions of the theory are not – or cannot – be satisfied in the experiments (and in reality!). Qualitative arguments are used to explain the principal features of the results.

In § 2 the flow in a straight highly conducting wall duct situated in a non-uniform field is considered. The value of the conductance ratio $\Phi = \sigma_w t/\sigma a = 8{\cdot}742$ reflects the attempt to simulate flow in a perfectly conducting duct (σ_w = conductivity of duct wall whose thickness is t). These results therefore represent the opposite extreme to those in part 1 where $\Phi \equiv 0$ and in contrast to that case the present results indicate that the effects of the non-uniform field region are localized. Measurements of the streamwise velocity component in the non-uniform field region near the wall at right-angles to the field lines show that as as the flow moves towards the region of lower field strength it is retarded but then quickly accelerates to a value greater than the mean before decelerating to the mean value. This novel behaviour has been predicted by a very approximate perturbation theory for a perfectly conducting duct developed by Holroyd (1976). The same theory over-estimates the observed pressure drop and, in fact, the pressure distribution along the duct can be satisfactorily represented by using the results of Chang & Lundgren (1961) and assuming fully developed flow throughout.

The ducts used in the remaining experiments have walls for which $\Phi \approx 0{\cdot}1$ and, as is common in MHD literature, they will be referred to as thin-walled ducts. These studies were undertaken because of their immediate practical relevance to the designs of putative fusion reactor coolant circuits as envisaged by the United Kingdom Atomic Energy Authority which employ liquid lithium in stainless steel pipes as a heat transfer medium so that $\Phi \approx 0{\cdot}1$ (Hancox & Booth 1971). Simulation of such values of Φ in a laboratory is not a straightforward matter and so § 3.1 is devoted to a brief survey of the possible ways of achieving such low values of Φ and a description of the method evolved for this work.

The flow in a straight thin-walled duct situated in a non-uniform field is discussed in § 3.2. From measurements of the pressure distribution along the duct it can be inferred that the effects of the non-uniform field region decay within a distance of a few duct radii upstream and downstream of it. Again, a pressure drop due to the non-uniform field is present but it is larger than that predicted by assuming a fully developed flow which suggests the presence of a recirculating current flow centred on the non-uniform field region which in turn implies some distortion of the flow in that region similar to that observed in the non-conducting duct. The pressure drop is also larger than the value predicted by Holroyd & Walker's theory although that is only valid if $\Phi^{\frac{1}{2}} \ll 1$ which is not truly satisfied here.

In § 3.3 are described pressure and potential distributions along a thin-walled duct comprising two parallel straight limbs offset from each other by a shorter straight limb which meets them at right angles and is aligned parallel to the field lines of a uniform magnetic field. The disturbing effects of the offset on the flows in the parallel pipes (which are perpendicular to the field lines) decay over distances of about $6a$ from the offset and the behaviour of the flow in those regions can be reliably described in a qualitative manner. However, the behaviour of the flow in the offset is less certain. A novel flow structure is proposed in which the fluid hugs the walls. This is supported by an order-of-magnitude analysis by Hunt (private communication) which also suggests a relationship between the pressure drop across the offset and N. Experimental results indicate some agreement with this hypothesis.

Corresponding measurements for a thin-walled duct comprising two straight limbs meeting at 90° situated in a uniform transverse magnetic field are discussed in § 3.4. Since the two pipes are mitred together the cross-sectional area of the duct on its plane of symmetry passing through the join is some 40 % larger than that of each pipe. The effects of this area variation are surprisingly noticeable and appear to persist over distances of up to $10a$ upstream and downstream of the bend which are somewhat larger than the disturbance lengths observed in the previous case. The potential distributions point to two recirculating current systems upstream and downstream of the bend which (presumably) give rise to the pressure drop there whose magnitude is of the same order as that in the previous case although now there is no obvious variation with N.

Finally in § 4 the principal features of the work are reviewed in the light of part 1, associated theoretical studies and technology. Some possible future MHD duct flow research topics are mentioned.

The principal items of apparatus used in these experiments, namely the electromagnet, flow circuit, manometer, hot-film probes, probe traversing gear and support trolley for straight ducts are described in part I and Holroyd (1976). In addition they also explain how potential distributions at the duct wall were made and how pressure distributions were built up from individual experiments. These details will not be repeated here.

2. Flow in a straight highly conducting walled duct in a non-uniform magnetic field

2.1. *Related theoretical work*

In physical or qualitative terms the behaviour of the flow can be readily described. The high conductivity of the wall is sufficient to make it an almost equipotential surface and so the longitudinal potential gradients and associated current flows found in a non-conducting duct are now virtually eliminated. Therefore to a first approximation the flow could be regarded as being fully developed everywhere.

Despite the appealing simplicity of this physical model, an analysis of the flow is exceedingly complex even for the idealized case of perfectly conducting wall and an inertialess, inviscid flow, i.e. Φ, N, $M \gg 1$. By using the general equations for steady MHD duct flows when $M, N \gg 1$ derived by Kulikovskii (1968), the governing partial differential equation for the pressure (which is constant along a field line) in a perfectly conducting duct can be deduced (Holroyd 1976, appendix V), Ludford & Walker (1980, equation (24)) but solutions can only be *computed* for a specified magnetic field. It also follows from Kulikovskii's equations that the potential distribution $\phi \neq 0$ in the fluid; the equality is only true for uniform fields.

Holroyd (1976, chap. 7) developed an approximate perturbation analysis of the flow, again for the idealized case described above. Small departures of the magnetic field $\mathbf{B}$ from a uniform value were assumed to result in corresponding departures from the fully developed flow values of the core flow variables, i.e. if

$$\mathbf{B} = (0, B_y^{(0)}, 0) - \alpha(B_x^{(1)}, B_y^{(1)}, 0) - \ldots$$

(since the magnetic field had no z component in this case) then, for example, the velocity $\mathbf{v} = v_x^{(0)} - \alpha(v_x^{(1)}, v_y^{(1)}, v_z^{(1)}) - \ldots$, etc., where α is small and the superscript (0) indicates the fully developed flow value. The analysis shows that in general $\phi = O(\alpha^2)$ and a detailed solution depends on the form of $\mathbf{B}$. The behaviour of v_x, the streamwise velocity component, in the non-uniform field region may be inferred from the form of $B_y(x, y)$ there which is sketched in figure 1 (*a*) (the regions of transverse uniform field are parallel to the Oy direction). By taking the curl of the momentum equation for an inviscid, inertialess core flow (Holroyd & Walker 1978, equation (2.3)) it follows that $\mathbf{B} . \nabla j_z = 0$ so that the value of j_z does not vary along a field line. From Ohm's law (Holroyd & Walker 1978, equation (2.1*b*)) $j_z = v_n B$ correct to $O(\alpha)$ here, where v_n is the velocity component perpendicular to a field line and parallel to the Oxy plane and so along a field line $v_n \propto B^{-1}$. Therefore, from figure 1 (*a*) it can be seen that, on moving along a magnetic field line from $y = 0$ to $y = a$, B decreases and hence v_n increases when $x > 0$ while for $x < 0$ the reverse is true and so v_x must take the form sketched in figure 1 (*b*). The associated variations in $v_y^{(1)}$ are of the same order as those in $v_x^{(1)}$ and exhibit a maximum at $x \approx 0{\cdot}2$ and two minima upstream and downstream of it (for $y > 0$) while the form of $v_z^{(1)}$ ($z > 0$) is similar to that of $-v_y^{(1)}$ ($y > 0$) but the variations are smaller by an order of magnitude, which implies that the flow is almost two-dimensional in the Oxy plane. These results show that the disturbance to the flow is confined to the non-uniform field region.

Walker & Ludford (1974) have analysed the flow in a perfectly conducting straight pipe and conical diffuser situated in a uniform transverse field. They found that the flow comprises a fully developed flow in the straight pipe and a quasi-developed flow

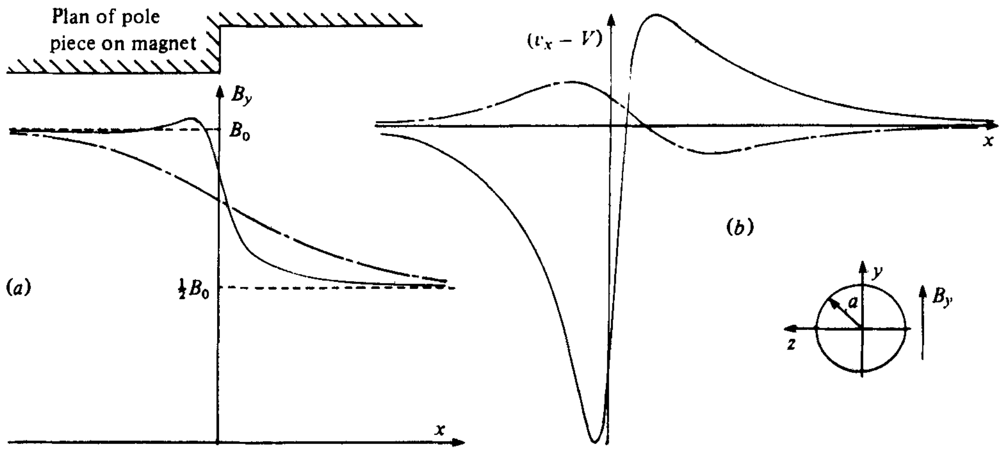

FIGURE 1. (*a*) Sketch of variation in flux density in non-uniform field region. (*b*) Sketch of streamwise velocity variation in non-uniform field region in highly conducting wall duct on plane $z = 0$. ——, near $y = a$; – – –, near $y = 0$.

in the diffuser which do not match at the join and two superimposed disturbance flows which although discontinuous themselves at the join do ensure a smooth transition between the developed flows. These disturbance flows decay exponentially in the straight pipe and algebraically in the diffuser. Although there is no variation in the velocity profile along a field line (which follows from the reasoning in the preceding paragraph) there is a strong dependence on the z co-ordinate; as the divergence or convergence of the diffuser increases so does v_x on the Oxy centre-plane.

The low values of ϕ allow much larger currents to flow than in a non-conducting duct and so the condition for an inertialess flow here is now $N \gg 1$.

2.2. *Apparatus, experiments and results*

The duct comprised a 1·45 m length of 82·55 mm internal diameter × 6·35 mm wall ($3\frac{1}{4}$ in. internal diameter × $\frac{1}{4}$ in. wall) highly-conducting copper tube for which $\Phi = 8{\cdot}742$. It was fitted with 14 pressure tap ports along the central 1·27 m of its length and at $0{\cdot}98\,\text{m} = 23{\cdot}8a$ from the inlet end provision was made for inserting a hot-film probe into the flow. Apart from its high conductivity, copper is eminently suitable in this case because an extremely low resistance amalgam forms at the copper–mercury interface (Baylis & Hunt, 1971, quote a resistance of $10^{-10}\,\Omega\,\text{m}^{-2}$) which allows free passage of current from fluid to wall and vice versa. Maximum attainable values of M and Re were 610 and 10 000 respectively.

Measurements of the streamwise velocity were made with a hot-film probe at the point $y/a = 0{\cdot}86$, $z/a = 0$, where the perturbation theory predicts very small values of v_y and maximum variations in v_x from the mean. The former point is important because the sensor is aligned parallel to the Oy direction and large velocity components parallel to it can seriously affect the reading (Malcolm 1969). Results from three experiments with $M = 603$, $N = 96{\cdot}7$ and $Re = 3765$ are summarized in figure 2. For comparison the appropriate theoretical distribution of v_x/V deduced from the perturbation theory with $\alpha = 0{\cdot}375$ is added (reasons for this value of α will be given below). It can be seen that apart from four points there is good qualitative agreement

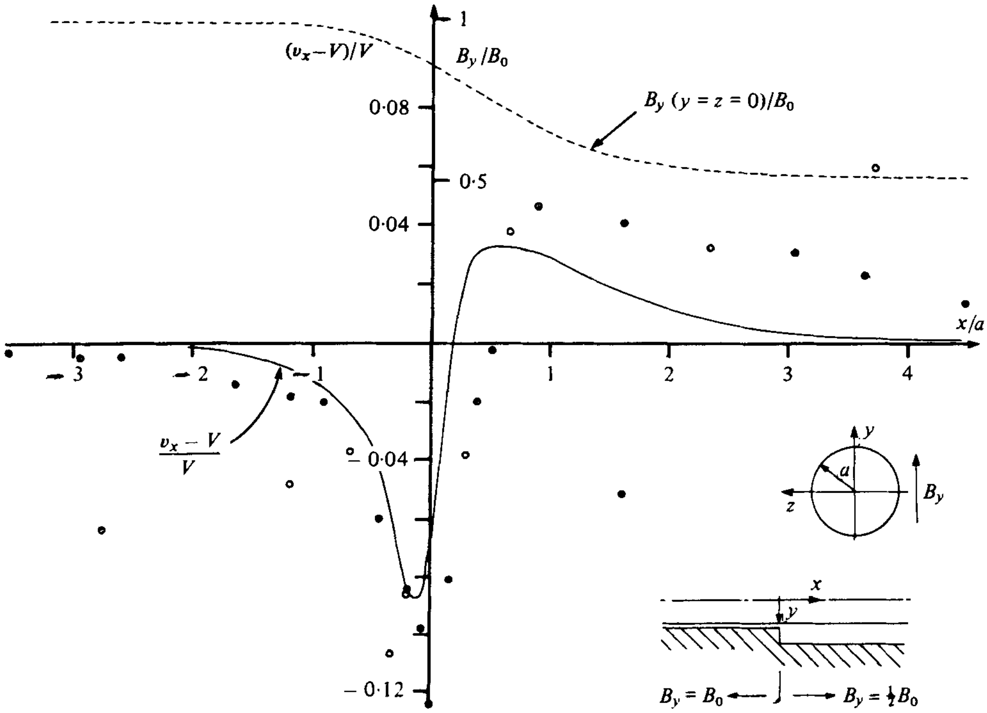

FIGURE 2. Measured streamwise velocity at $y = 0{\cdot}86a$, $z = 0$ in non-uniform field region in highly conducting wall duct. Different symbols indicate results from different experiments but all with $M = 603$, $N = 96$, $Re = 3765$. The solid curve represents the theoretical value predicted by the approximate perturbation analysis of Holroyd (1976). Variation of B_y along central axis of duct is indicated by the dashed curve.

between measured and predicted values, but it must be borne in mind that the latter depends on the value of α. The results suggest a more persistent disturbance to the flow than is predicted; this could be due to the fact that the theory assumes $\Phi \to \infty$, which is not true in the experiment. Of the four odd points, the two for $x < 0$ were the first two of one experiment and it is probable that the hot-film probe anemometry equipment had not warmed up properly. The other two odd points were the last of two experiments and their inconsistency with the majority of the readings is mystifying – one possible explanation is that accumulated scum on the sensor suddenly broke off, thereby affecting the readings.

Calibration of the probe before these experiments had been carried out by moving the duct and probe so that the latter was in the high field strength region of the magnet, remote from the non-uniform field regions at $x \approx 0$ and $x \approx -10{\cdot}88a$ (the upstream end of the magnet). In this position it was assumed that the velocity profile would be uniform and hence known and this assumption was justified by the results.

For a more detailed discussion of these measurements with hot-film probes together with those mentioned in part 1 see Holroyd (1980).

The pressure distribution along the duct, comprising five sets of results with the duct in different positions with respect to the magnetic field, is shown in figure 3. In this duct the currents and hence the pressure gradients are larger than those in a non-conducting duct by a factor of M and so reliable measurements could be made even

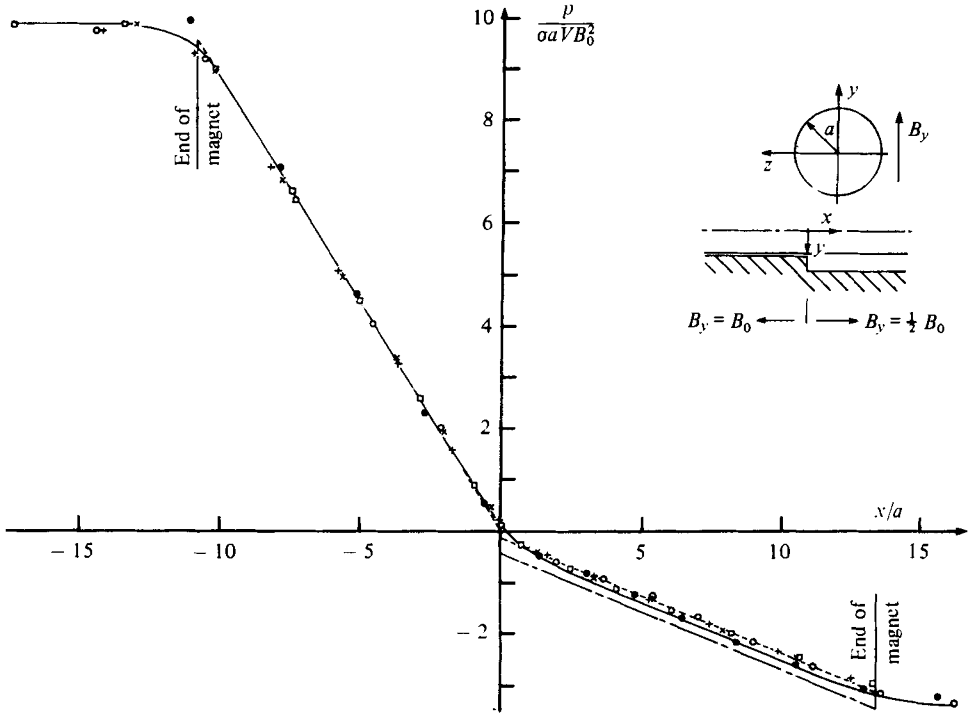

FIGURE 3. Pressure distribution along highly conducting wall duct, $\Phi = 8{\cdot}742$. ———, pressure distribution derived from theory of Chang & Lundgren (1961) and assuming a fully developed flow throughout, $p/\sigma a V B_0^2 = -\Phi(1+\Phi)^{-1}\int (B/B_0)^2\, d(x/a)$; – – –, mean asymptote through experimental points in region of low uniform field strength; ———, low field strength asymptote from perturbation theory of Holroyd (1976) with slope decreased by $\Phi(1+\Phi)^{-1}$ to account for conductivity of duct wall. (M, N, Re): □, (608·2, 119·20, 3103); ○, (577·9, 103·24, 3235); +, (552·4, 67·23, 4539); ×, (524·0, 60·85, 4524); ●, (301·0, 9·47, 9564).

at very low velocities and field strengths. Pressures were measured for $310 < M < 608$ and $9{\cdot}47 < N < 119{\cdot}2$ and are plotted in non-dimensional form as $p/\sigma a V B_0^2$. Although there is, perhaps, some suggestion of disagreement between the distributions in the region of low field strength there is no systematic variation from the mean asymptote which is indicated by the dashed line (determined visually). The continuous curve represents the pressure distribution derived by assuming a fully developed flow throughout and using the results of Chang & Lundgren (1961) so that

$$p/\sigma a V B_0^2 = -\Phi(1+\Phi)^{-1}\int (B_y/B_0)^2\, d(x/a).$$

It can be seen that for $x > 0$ this latter distribution lies *below* the mean experimental asymptote and therefore overestimates the pressures there by $0{\cdot}13\sigma a V B_0^2$. In fact this discrepancy varies with the particular B_y distribution employed in the above integral. Since $dp/dx \propto v_x B_y^2$ (neglecting the effects of the extremely small electric field $\nabla\phi$) it is clear that the optimum B_y distribution would be at a level where the difference between v_x and V throughout the non-uniform field region is minimized. Here, the perturbation analysis suggests that $y \approx 0{\cdot}5a$ is a suitable level. The curve in figure 3 is computed for values of B_y measured on the central axis of the duct, i.e. $y = z = 0$. Use of distributions along other axes shows that the mean experimental slope in the low field strength region can be approached more closely. Of course, in many situations where duct and field geometries are more complex such refinements will not be possible. It is hardly surprising to find that the perturbation theory overestimates the mean experiment distributions in the low field strength region, in this case by $0{\cdot}4\sigma a VB_0^2$. The value of α used in the calculations for this theory, namely 0·375, ensured that the correct pressure gradients would be realized in each uniform field region. In addition, to allow for the finite conductivity of the duct wall these theoretical pressures are reduced by a factor of $\Phi(1+\Phi)^{-1}$ which follows from Chang & Lundgren's theory.

These results show that in a highly conducting wall duct the effects of a non-uniform magnetic field are confined to that region. Internally there is slight distortion of the flow, which to a first approximation can be assumed to be fully developed. All this is in marked contrast to the flow in a non-conducting duct where the same non-uniform field affects the flow over large $O(aM^{\frac{1}{2}})$ lengths upstream and downstream and introduces a large pressure drop of $O(aM^{\frac{1}{2}} \times$ fully developed flow pressure gradient) across the non-uniform field region.

3. Flow in thin-walled ducts

3.1. *Laboratory realizations of thin-walled ducts*

Flows in thin-walled ducts are of greater importance, perhaps, than flows in non-conducting or highly conducting wall ducts since the latter are idealizations unlikely to occur frequently in practice. In fusion reactor technology values of Φ ranging from 0·1 (Hancox & Booth 1971) to 0·01 and below (Hoffman & Carlson 1971) have been quoted along with typical pipe wall thickness t of $O(\frac{1}{10}a)$. Such low values of Φ can be realized by using unusual fluid/pipe wall combinations; for example Carlson (1974) used (molten) lithium in stainless-steel pipe to give $\Phi \approx 0{\cdot}05$ and $t/a \approx 0{\cdot}15$, but against the use of such liquids are the restrictions and difficulties they impose on the associated apparatus. In any case, the only liquid readily available for the present work was mercury. Ihara, Tajima & Matsushima (1967) used mercury in thick-walled small-bore carbon tubes ($a = 1{\cdot}5$ mm, $t \approx a$, $\Phi \approx 0{\cdot}05$) but whether larger tubes with thinner walls could be employed is open to question. The same authors also comment on the problems of using mercury in stainless steel tubes due to high yet variable resistance at the fluid–wall interface.

In this work low values of Φ were achieved by using an extremely thin (about 0·1 mm) copper wall. Ducts were fabricated in two ways.

(i) A length of the 71·85 mm bore PVC tube as used in the non-conducting duct experiments was slit longitudinally and each half lined internally with 0·125 mm copper shim held in place by Evo-stik. The two halves were clamped together with non-magnetic Jubilee clips with O-ring cord along the joint to prevent leaks. In addition, special copper bolts were used to form pressure tap ports which prevented mercury leaking behind the copper shim (see figure 4a). This method of construction has the advantages of being cheap and relatively straightforward but the duct so formed can only be used when the plane of the breaks in the wall is at right angles to and coincident with the plane of symmetry of the magnetic field lines because in that position the current flow in the walls is zero at the breaks (see figure 4a).

(ii) A plastic, slightly tapered (about 0·05°), circular mandrel was electroplated with copper which was ground back slightly to ensure an even plating thickness all round. After lapping the copper with fibreglass the mandrel was removed, leaving an internally

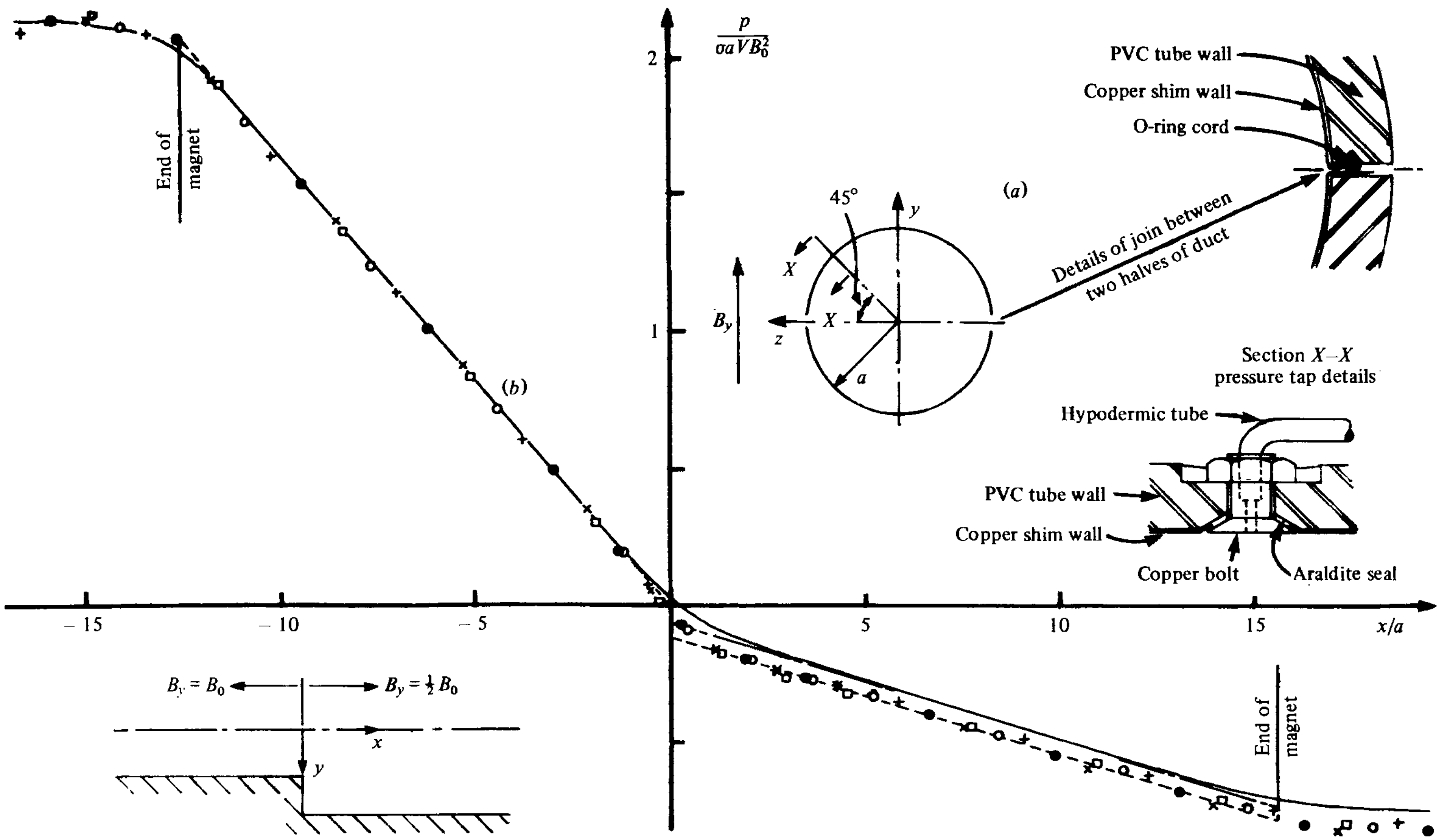

FIGURE 4. Thin-walled duct, $\Phi = 0{\cdot}2$. (a) Some constructional details. (b) Pressure distribution. ——, pressure distribution derived from theory of Chang & Lundgren (1961) and assuming a fully developed flow; – – –, mean asymptote through experimental points in region of low field strength; ————, low field strength asymptote derived from theory of Holroyd & Walker (1978) with slope decreased by a factor of $(1+\Phi)^{-1}$ which is neglected in their analysis valid for $\Phi^{\frac{1}{2}} \ll 1$. (M, N, Re): ×, (526, 49·8, 5540); +, (518, 42·0, 6388); ●, (496, 37·8, 6519); ○, (474, 22·6, 9944); □, (363, 13·2, 9953).

plated duct without any breaks in the wall. An additional advantage of this method is that plated elbows etc. can be formed which can be used in conjunction with plated tubes to assemble ducts with bends.

It is well known that mercury dissolves copper but in these experiments there was no indication of serious erosion of the duct walls. In fact, the solubility data quoted by Hansen (1958) implies that the mercury in the flow circuit (about 150 kg) could only take up an amount of copper equivalent to about 2 % of the thickness of a duct wall.† On this basis, the claim by Baylis & Hunt (1971) that the amalgam layer on the copper walls of their annular duct thickened by 25 μm during a short experiment must be viewed with scepticism. Furthermore, the predictions by Glabersen, Donnelly & Roberts (1968) of rapid erosion of copper walls and of large and unpredictable resistance to current flow at a mercury–copper interface(!) are refuted. By using strips of copper shim it can be shown that the formation of an amalgam layer on one surface decreases the conductance of the strip by about 1 % but thereafter over a period of days there is no further significant decrease. This observation was borne out by the experiments with these ducts which lasted for periods of a few weeks. However, it was subsequently discovered that over a period of 16 months the conductance ratio decreased by about 13 %.

3.2. *Experiments with a straight duct in a non-uniform magnetic field*

The duct for these experiments was made using the type (i) construction method described above. Its mean radius $a = 35{\cdot}78$ mm so that $\Phi = 0{\cdot}2$ while its length was $1{\cdot}5\,\text{m} = 42a$. Fifteen pressure tap ports were set out over the central 1·38 m of its length. Pressure measurements taken with the duct in different positions with respect to the magnet are summarized in figure 4 (*b*) and it can be seen that the mean pressure distribution is independent of the range of values of M and N used in the experiments, namely 363–526 and 13·2–49·8 respectively. At lower values of Φ satisfying $\Phi^{\frac{1}{2}} \ll 1$ theory predicts that the flows in thin-walled and non-conducting ducts are qualitatively the same (Holroyd & Walker 1978); in particular, a longitudinal recirculating current flow centred on the non-uniform field region and extending $O(a\Phi^{-\frac{1}{2}})$ distances upstream and downstream introduces a pressure drop of $O(a\Phi^{-\frac{1}{2}} \times$ fully developed flow pressure gradient) into the pressure distribution. In figure 4 (*b*) the dashed line represents the mean asymptote of the pressure distribution in the low field strength region and it can be seen that it lies below that derived from Chang & Lundgren's theory and assuming a fully-developed flow ($p/\sigma a V B_0^2 = \Phi(1+\Phi^{-1}) \int (B_y/B_0)^2\, d(x/a)$). Therefore, although the large value of $\Phi^{\frac{1}{2}}$ ($= 0{\cdot}447$) here invalidates Holroyd & Walker's detailed results, a recirculating current flow must exist. In fact, in this particular case the asymptotes of Holroyd & Walker's predicted pressure distribution, reduced by a factor of $(1+\Phi)^{-1}$ (which is neglected in their analysis), which are indicated by the middle lines in figure 4 (*b*), coincide with those of the continuous curve. The discrepancy between the predicted and measured pressure distributions in the low field strength regions Δp, is given by $\Delta p/\sigma a V B_0^2 = 0{\cdot}053$, which on the expanded scale employed by Holroyd & Walker, namely $\Delta p \Phi^{-\frac{1}{2}}/\sigma a V B_0^2 = 0{\cdot}119$.

† Unless, as pointed out by J. T. D. Mitchell (private communication), there are temperature gradients in the flow circuit, in which case copper will be deposited out of solution since its solubility in mercury varies with temperature.

The other important feature of figure 4 (*b*) is that there appears to be little disturbance to the flow upstream and downstream of the non-uniform field region.

3.3. *Flow in a* ——┌—— *tube in a uniform field*

Using the type (i) construction method a duct was fabricated from two half-metre lengths of 38 mm $= 2a$ bore tube, two 90° elbows and a short cross-piece such that the two longer tubes were offset by 149 mm $= 7{\cdot}86a$. The thickness of the copper wall was 0·05 mm so that $\Phi = 0{\cdot}155$. All pole pieces of the electromagnet were removed, leaving a 200 mm air gap which just accommodated the duct with the offset perpendicular to the faces of the yoke. Because of fringing effects the flux density across the air gap varied as shown in figures 5 (*a*) and (*b*). Therefore the field lines were not quite parallel to the wall of the offset wall while across the long tubes the transverse field strength varied from $1{\cdot}0326B_0$ at their nearest point to the yoke face to $0{\cdot}883B_0$ at a diametrally opposite point, where B_0 is the average flux density over the cross-section of that tube. Although these variations could have been reduced by attaching shims to the yoke there would then have been some reduction in overall field strength. All experiments were carried out at the highest possible Hartman number, $M = 227{\cdot}5$, while the maximum Reynolds number was 20 000.

Some features of the internal structure of the flow may be deduced from measurements of the potential distribution at points on the wall at which the field lines are tangential, i.e. the point $y = 0$, $z = -a$ on the tube in figure 5 (*a*) and corresponding points elsewhere along the duct. A typical set of results is shown in figure 5 (*c*), where the potential relative to the central axis of the duct ϕ is plotted in non-dimensional form as ϕ/aVB_0, values being recorded at each of the 15 pressure tap ports along the duct. These results were measured for $M = 207{\cdot}1$, $N = 27{\cdot}33$ and $Re = 1888$ but other results with $4{\cdot}9 < N < 87{\cdot}2$ showed no significant difference. An important feature of these results is that they are symmetric (to within experimental error) about the mid-point of the duct, which indicates that the nonlinearizing effects of inertial stresses are negligible in this case. The close agreement of the measured potential distribution with the fully developed flow value over most of both long members of the duct indicates that (i) the effects of entrance and exit flows and the adjacent non-uniform fields at each end of the magnet are quickly suppressed and, more importantly, (ii) the effects of the offset are felt over distrances of about $6a$ upstream and downstream. Over this distance of $6a$, ϕ falls to about 50 % of its fully developed flow value and eventually falls to zero at the mid-point of the offset. This suggests that two recirculating current systems in the fluid and wall centred on each elbow are set up; possible current streamlines of the former are sketched in figure 6 (*a*). Upstream and downstream of the offset the pressure gradient will therefore be increased but in the offset itself the pressure gradient can only be due to viscous and inertial effects and will be small – in fact the ratio of the pressure gradients for a fully developed parabolic velocity profile in an aligned uniform field and a fully developed flow in a thin-walled duct in an equal uniform transverse field is $8(1+\Phi)/M^2\Phi$. A sketch of the observed pressure distribution in and near to the offset is shown in figure 6 (*b*); in the offset the pressure gradient was extremely small and there was some indication of nonlinear pressure distributions upstream and downstream of it. In addition there was a net pressure drop across the offset Δp which will be discussed in the next paragraph.

The behaviour of the flow in the offset is not easily resolved. Upstream of it the

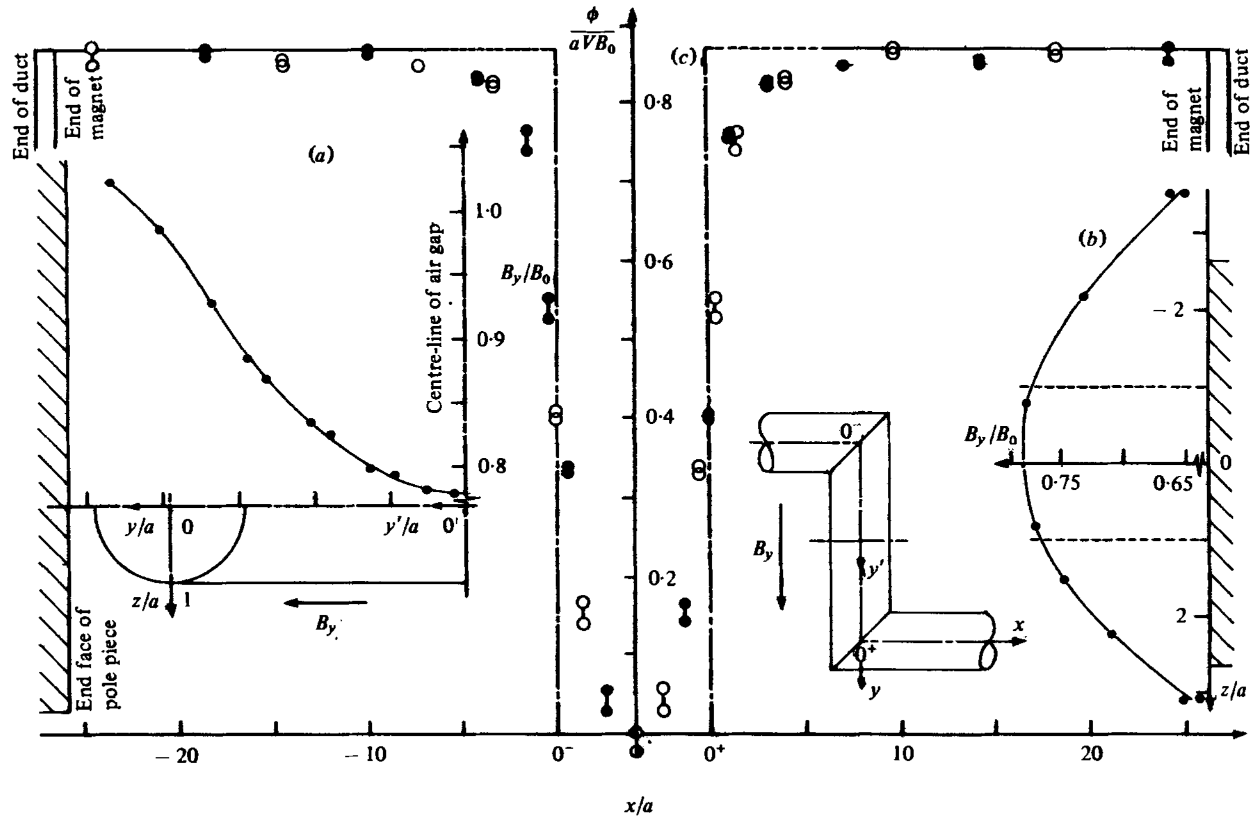

FIGURE 5. Experiments with ——⌐—— duct. (*a*) Variation of B_y with y' on plane $z = 0$. (*b*) Variation of B_y with z on centre-plane $y' = 0$ of air gap. (*c*) Potential distribution $\Phi = 0{\cdot}155$, $M = 227{\cdot}1$, $N = 27{\cdot}33$, $Re = 1888$. The two symbols at each point indicate readings with respect to different reference points. ●—● are points measured with flow from left to right; ○—○ are same points plotted in reverse order to demonstrate symmetrical nature of distribution about mid-point of duct.

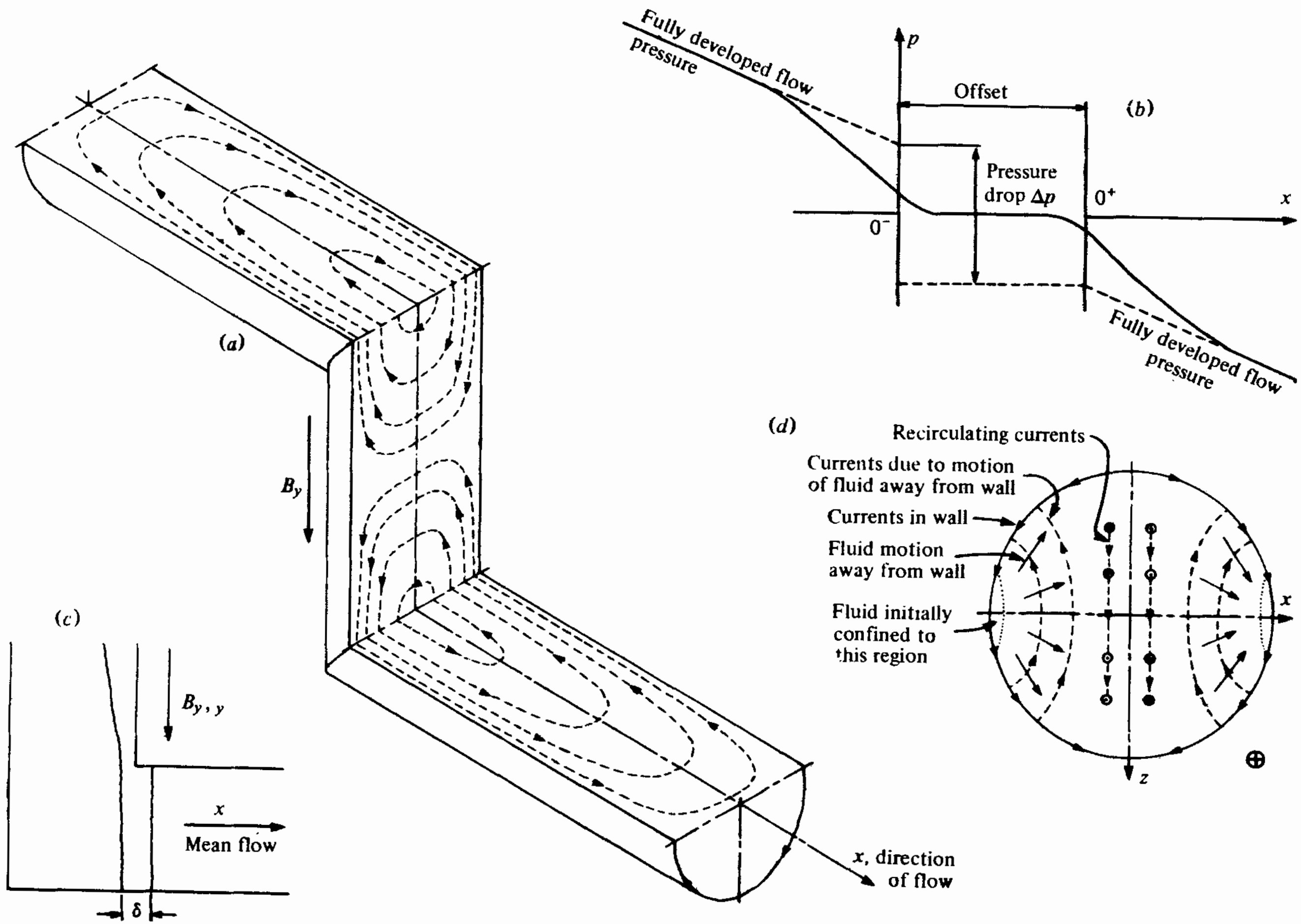

FIGURE 6. Flow in ——┌—— duct. (a) Sketch of recirculating current loops in fluid in and near offset. (b) Sketch of pressure distribution in and near offset. (c) Notation for Hunt's order-of-magnitude analysis of flow near junction of straight pipe and offset (see main text). (d) Sketch of suggested fluid and current flows in offset. Left-hand half of figure shows flow near entry to offset; right-hand half shows flow near exit.

available theoretical evidence (Walker & Ludford 1974; Holroyd & Walker 1978) and the present results suggest that fluid migrates towards the *Oxy* centre-plane of the duct with the process acting in reverse downstream (axes are those defined in figure 5). For lower values of Φ (satisfying $\Phi^{\frac{1}{2}} \ll 1$) Holroyd & Walker's theory suggests that at each end of the offset at least the flow is confined to jets near the walls at $z = \pm a$ (this may be deduced by considering the flow in an offset which meets the tubes at right angles to the field lines at an angle $\theta \neq 90°$; then the streamlines must lie on surfaces for which $\int dy =$ constant (since $B_0 =$ constant here); as $\theta \to 90°$ these surfaces, and hence the flow, are squeezed into the boundary layers at $z = \pm a$). However even if a less extreme form of this velocity distribution did exist in this case, it is difficult to incorporate the associated current flows into an overall picture of the flow which satisfies all the relevant equations and boundary conditions. On the other hand, it has been shown that at a sudden increase in cross-sectional area of a duct in a uniform transverse field the flow can be confined to boundary layers (Hunt & Ludford 1968). Order-of-magnitude arguments by Hunt (private communication) point to the initial flow in the offset being confined to a shear layer of thickness $O(aN^{-\frac{1}{3}})$ at the mouth of the upstream part of the duct.† On this basis it is possible to construct the flow pattern sketched in figure 6(*d*) in the offset; the flow enters the offset confined to a narrow region close to the mouth of the upstream part of the duct; as the fluid moves down the offset some of the fluid moves towards the centre of the pipe while the rest moves round the walls – the arrows in the left-hand half of figure 6(*d*) indicate this motion. In the lower half of the offset this process is reversed so that the fluid reaches the mouth of the downstream pipe in a narrow region on the opposite wall to that from which it started (see right-hand half of figure 6*d*). The current flows associated with this motion, also indicated in figure 6(*d*), are not inconsistent with the current flows in the conducting walls implied by the observed potential distribution.

A further prediction of Hunt's order-of-magnitude analysis is that across each thin layer at the mouth of each pipe there is a pressure drop Δp such that $\Delta p/\sigma a V B_0^2$ varies as $N^{-\frac{1}{3}}$. Measured pressure drops across the offset derived from pressure distributions along the duct are plotted in figure 7 as $\Delta p/\sigma a V B_0^2$ against $N^{-\frac{1}{3}}$ and it can be seen that there is some indication that indeed $\Delta p/\sigma a V B_0^2 = c + N^{-\frac{1}{3}} d$, where c and d are constants.

Indirect support for these results comes from Branover *et al.* (1966) who showed that there is a pressure drop of $O(\sigma a V B_0^2 N^{-\frac{1}{3}})$ at the mouth of a non-conducting Pitot tube in a transverse field for $N > 0{\cdot}3$ (based on the probe dimensions) and an associated shear layer, parallel to the field lines, of thickness $O(aN^{-\frac{1}{3}})$.

Attractive though this inertial model of the flow may appear, it must be stressed

† Hunt's reasoning runs as follows: referring to figure 6(*c*) it follows that in the thin shear layer of thickness δ at the mouth of the tube joining the offset $v_y \approx aV/\delta$, $v_z \approx aV/\delta$ and $j_z \approx \sigma V B_0$. Relevant terms of the vorticity equation, i.e. the curl of the momentum equation [see Holroyd & Walker 1978, equation (2.1*a*)] are

$$\rho \frac{\partial}{\partial x}\left(v_y \frac{\partial v_y}{\partial y} + v_x \frac{\partial v_y}{\partial x} + \dots\right) = -B_0 \frac{\partial j_z}{\partial y}.$$

The left-hand side $\approx \rho V^2 a/\delta^3$ and the right-hand side $\approx \sigma V B_0^2/a$ and so it follows that $\delta \approx aN^{-\frac{1}{3}}$. From the momentum equation the ratio of electromagnetic and inertial terms in this layer $\approx \sigma V B_0^2/(\rho V^2/\delta) \approx N^{\frac{2}{3}}$ and so the latter are negligible. Therefore to complete the balance of forces across this layer there must be a pressure drop $\Delta p \approx \sigma V B_0^2 \delta \approx \sigma a V B_0^2 N^{-\frac{1}{3}}$, i.e. the non-dimensional pressure drop $\Delta p/\sigma a V B_0^2 \approx N^{-\frac{1}{3}}$.

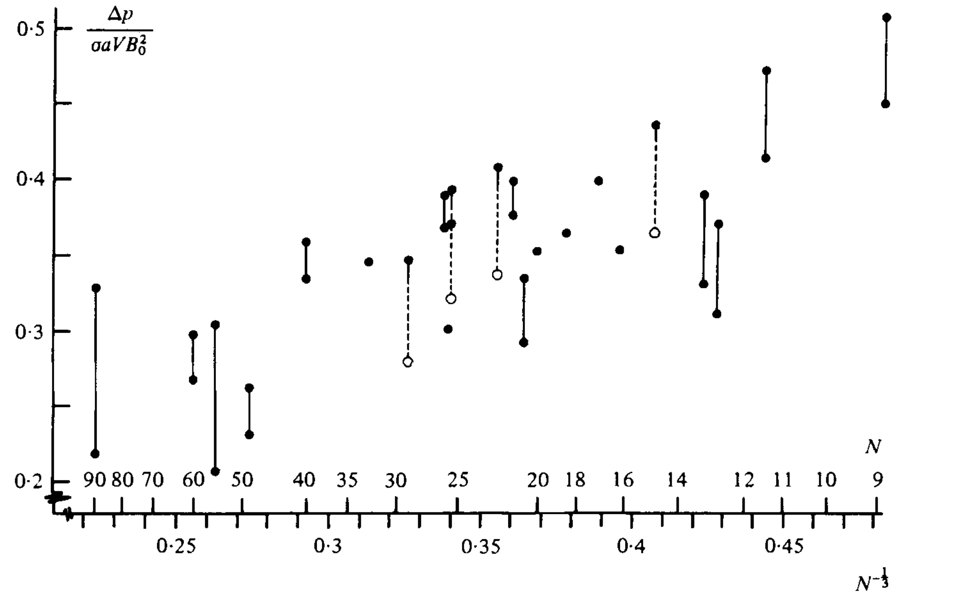

FIGURE 7. Variation of pressure drop Δp across offset in ——⌐—— duct with $N^{-\frac{1}{3}}$. In most cases it was only possible to determine a range of values for Δp which are indicated by ○——○. If the results suggested that the exact value of Δp might lie nearer the upper (or lower) limit of such a range then the range is indicated by ●— – – ○ with the ●— part of the symbol indicating the more probable values of Δp.

that other models might be constructed. The flow structure it predicts differs radically – perhaps too radically – from that predicted by the Holroyd–Walker inertialess model, which suggests that the true flow is probably more complex and contains features of both models. Clearly further experiments and theoretical studies are necessary to elucidate this problem.

This experiment is closely related to the study of flow through a 180° non-conducting bend carried out by Bocheninski, Tananaev & Yakovlev (1977). Effectively the offset and two elbows in this duct have been replaced by a semi-circular bend and, of course, the downstream straight pipe runs in the opposite direction. Their measurements of the pressure drop across the bend satisfy a relationship of the form

$$\Delta p/\sigma a V B_0^2 = 0{\cdot}125 + 0{\cdot}5/N$$

for $M = 360$ and $12 < N < 100$, which disagrees with the above ideas. However, their measured pressure drop includes not only that across the bend but also pressure drops along the straight tubes upstream and downstream of the bend and it is quite clear from other results which they present that in many cases fully developed flow is not established in those pipes due to the effects of the non-uniform field where they cross the edge of the magnet and also the effects of the bend itself. Consequently their results must be viewed with suspicion.

3.4. *Flow in a 90° ∨ tube in a uniform field*

This duct was built up in a similar manner to the one described in § 3.3, namely from two half-metre straight tubes and a 90° elbow. Again the bore was 38 mm but the plating thickness was 0·04 mm so that $\Phi = 0{\cdot}118$. The necessarily large area of uniform transverse field was produced by adding to the magnet pole pieces whose cross-section is sketched in figure 8 (*a*). The field was uniform to within 5 % over the central 90 % of the pole face with a maximum value of 0·383*T* and so the corresponding Hartmann number, which was used in all the experiments, was 188.

A typical distribution of potential measured at the 15 pressure tap ports on the outside wall of the bend for $M = 188{\cdot}4$, $N = 25{\cdot}9$ and $Re = 1369$ is shown in figure 8 (*b*). As in the —⌐— duct, the distribution is symmetric about the mid-point of the bend, indicating that inertial effects are negligible. Other results at the same value of M but with $3 < N < 59$ showed no significant difference except that symmetry was less obvious at the lowest value of N. It can be seen from figure 8 (*b*) that although there is only a modest decrease in the potential ϕ at the mid-point of the bend (about 10 % of the fully developed flow value) the effect is felt over distances of about $9a$ away from the bend. A puzzling feature of the results is that over the central part of each arm of the duct the measured potential is greater than expected by up to 5 %. One possible reason for this is that the velocity profile is non-uniform whereas in fully developed flow it should be uniform. From Ohm's law,

$$\phi = -\frac{1}{\sigma}\int_{-a}^{0} j_z dz + B_0 \int_{-a}^{0} v_x dz;$$

as will be shown later the first term on the right-hand side will be negative but the second will always be positive with a value depending on the form of $v_x(z)$. If fluid hugs the walls near $z = \pm a$ then the circular cross-section of the duct ensures that

$$\int_{-a}^{0} v_x dz > \int_{-a}^{0} V dz;$$

this could account for the increase in ϕ. When $\Phi^{\frac{1}{2}} \ll 1$, Holroyd & Walker (1978) predict that such velocity profiles can exist over distances of $O(a\Phi^{-\frac{1}{2}})$ from a non-uniform field region. However, here where $\Phi^{\frac{1}{2}} = 0{\cdot}344$ the persistence of such a profile, if it exists, for a distance of $15a$ would be unusual. The low values of ϕ at each end of the duct are due to the decrease in B_y in those regions. In the —⌐— duct entry and exit effects were quickly suppressed. This may reflect the way that the flow was led to and from each duct; in the —⌐— duct it entered and left via co-axial diffusers connecting to 12·5 mm bore pipe while here that same small-bore tube was connected directly to the duct so that its axis made an angle of about 45° with that of the tube.

That the effects of the bend should be expressed over slightly longer distances than those due to the offset in the previous experiments is, perhaps, surprising. Even though there is a slight increase in cross-sectional area of the duct at the bend (see figure 8*c*) there is no obvious reason for any dramatic change in the internal structure of the flow. Indeed, for smaller values of Φ (such that $\Phi^{\frac{1}{2}} \ll 1$) the theory of Holroyd & Walker (1978) implies that the streamlines in each arm of the duct would be parallel to its axis and meet at the mid-point of the bend at right angles. Furthermore, the asymmetric arrangement of the flow would eliminate any variations in ϕ along the duct. However, the present results point to two recirculating current flows being set up as shown in figure 8 (*c*). Associated with them will be a pressure drop across the bend which was observed in measured pressure distributions for $M = 188$ and $4{\cdot}3 < N < 30$. The value of the pressure drop was of the same order as that in the duct and had an

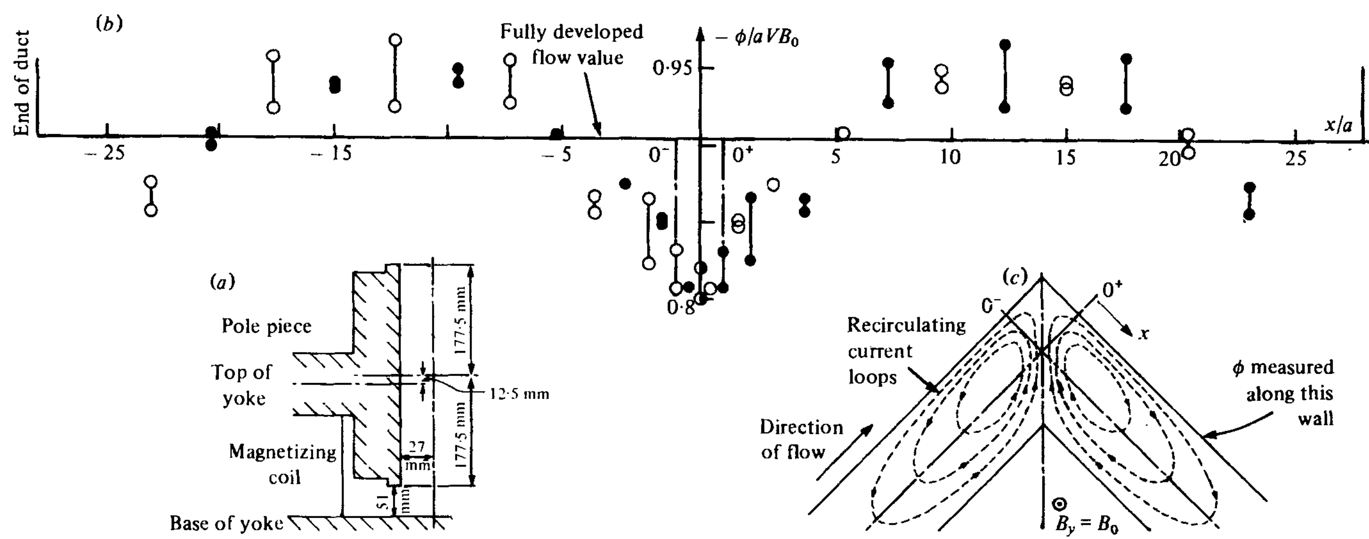

FIGURE 8. (*a*) End elevation of pole pieces attached to magnet to produce large area of uniform for ∨ duct experiments. (*b*) Potential distribution along outside of bend in ∨ thin-walled duct, $\Phi = 0{\cdot}118$, for $M = 188{\cdot}4$, $N = 25{\cdot}9$, $Re = 1369$. The two symbols at each point indicate readings with respect to different reference ports. ●● are points measured with flow from left to right; ○○ are same points plotted in reverse order to demonstrate symmetrical nature of distribution about mid-point of duct. Distances are measured from intersection of centre-lines of the two long arms of the duct. (*c*) Sketch of recirculating current flows in fluid near bend in ∨ duct.

average value of $0{\cdot}5\sigma a VB_0^2 \approx 0{\cdot}5(1+\Phi)\,\Phi^{-1}a \times$ (fully developed flow pressure gradient) but there was no obvious variation with N.

If these effects near the bend are due to the changes in area then it is quite possible that they could be reduced by a differently shaped bend, e.g. one of constant circular cross-section formed by rotating a circle about a tangent.

Bocheninskii *et al.* (1977) also measured pressure distributions along their non-conducting duct with a 180° semi-circular bend when its plane was perpendicular to the field lines. Their results for $M = 360$ and $N = 2$ extend for distance of $6a$ upstream and $15a$ downstream of the bend and there is no indication of a fully developed flow pressure gradient in either region. Consequently the pressure drop across the bend, $0{\cdot}12\sigma a VB_0^2 \approx 0{\cdot}1aM \times$ (fully developed flow pressure gradient), must underestimate the true value. Even so, it can be seen that the drop is substantial compared with that observed in the present duct. Furthermore, the pressure distribution around the bend is non-symmetric and there is a maximum pressure difference of about $0{\cdot}35\sigma a VB_0^2$ between the outside and inside walls of the bend; these features show the significance of inertial effects.

4. Conclusions and discussion

These experiments with straight ducts situated in a transverse non-uniform magnetic field, together with those with non-conducting ducts in part 1, show that as the conductivity of the duct wall increases the effects associated with the non-uniform field are suppressed. Effectively, for $\Phi > 1$ the flow can be regarded as being fully developed to a good approximation. For $\Phi < 1$ a recirculating current flow along the duct centred on the non-uniform field region is established which introduces a pressure drop into the pressure distribution and affects the internal structure of the flow. While values of $\Phi \approx 0{\cdot}1$ appear to have practical relevance at the moment, analyses of flows in such ducts are conspicuous by their absence.

There is a puzzling fundamental difference in the flow in perfectly conducting wall ducts implied by the analyses of Walker & Ludford (1974) and Holroyd (1976). The former's analysis of flow in a conical diffuser in a uniform field indicates a strong dependence of the core flow variables on the z co-ordinate while the latter's approximate perturbation analysis of flow in a straight pipe in a non-uniform field reveals a flow almost independent of z (axes are as indicated in figure 3). Perhaps this is related to the three-dimensional nature of the diffuser and the (planar) two-dimensional form of the field in the two cases.

The experiments with ducts containing bends, which might be regarded as variable area ducts, in uniform magnetic fields show that the effects of bends are similar to those created by non-uniform fields. Because the ducts are symmetric about a mid-point, it is inferred that *two* recirculating current systems are set up which lead to pressure drops in the pressure distribution and, presumably, alter the internal structure of the flow upstream and downstream of the bend or offset. For the ——┘¯¯¯ duct a novel prediction relating the pressure drop across the offset to N has found reasonable support. However, a detailed analysis of the flow in the offset is clearly a non-trivial problem. Studies with the ∨ duct suggest that the shape of the bend might affect its disturbing influence on the flow.

On the whole the experimental results are consistent, if not in exact agreement, with theoretical predictions relating various features of the flows (e.g. the pressure drop across the non-uniform field region) to M, N and Φ. Such disagreements that have been observed can usually be attributed to restrictions on those parameters by the theory not being upheld in the experiments. However, this does not necessarily imply that such theoretical relationships are true at much larger values of M, etc., than those in the experiments. Increased justification for their acceptance can only be provided by agreement with other experimental data that becomes available.

From a practical point of view, the important consequence of this work is that neither fully developed MHD duct flows nor corresponding hydrodynamic duct flows are reliable guides to flows in ducts with complex shapes and/or when the magnetic field varies in space. Internal restructuring of the flow might be significant if heat transfer effects are being considered. More important, perhaps, are the pressure drops that can be introduced. Consider, for example, the problem of minimizing the pressure losses along a pipe. Without the magnetic field, the pipe may be chosen to withstand the highest internal pressure along its length. With the magnetic field, the pressure distribution is related to Φ and hence the local wall thickness t. Therefore there is an incentive to decrease t along the pipe as the pressure decreases (subject, of course, to other structural requirements). Even if it is not possible to vary t continuously along the pipe it may seem better, at first sight, to use several pipes of the same bore but with smaller wall thickness. However, the resulting variations in Φ along the duct are analogous to varying field strength and/or area problems which might lead to additional pressure drops being introduced. Furthermore, if the pipe and fluid form part of a heat transfer system it is possible to envisage a situation – albeit an extreme one – of a large temperature increase occurring over a short length of the pipe. This too would change the value of Φ.† Clearly, optimizing the local pressure/local Φ relationship is not a straightforward problem!

Future-engineering-type MHD duct flow problems worthy of study may be drawn from hydrodynamic duct flow studies. The flow in the ——┌—— duct here can be extended to flow in a duct with a bend such that a flow initially subjected to a transverse field becomes flow in a aligned field. A logical extension of this and variable area duct flows is flow near a bifurcation. Related to that is flow from several pipes into a header tank. In these last two examples the relative orientation of field and pipes will be important.

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† This possibility was suggested by J. T. D. Mitchell.

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